REGENERATIVE WEIGHT LIFTING EQUIPMENT: THE POWER LIFTER

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ABSTRACT:

The objective of this project was to produce energy efficiently from an untapped source of energy. The gym is where people congregate to lift heavy weights; while that comes with benefits for fitness, the power of the human body goes to waste. This project provides a solution to this issue by creating a non-intrusive method of energy production in a highly trafficked area. The non-intrusive constraint on energy production is satisfied through a one-way velocity activated clutch that would prevent any extra resistance throughout the workout. The efficiency of energy production is supported by a variable moment of inertia flywheel. The flywheel also stops the weights from slamming with high energy while maintaining a high efficiency for all of the machine's different weights. The results show that this system has an efficiency of an estimated 7%, regenerating 15.04 J of energy with a maximum instantaneous wattage of 40.79 W in a 45 pound pull. These statistics show that the system produces significant energy, achieving the ultimate goal of the project. With improved materials, this system can be a commercially viable way of regenerating otherwise wasted energy at the gym.

BACKGROUND:

Energy is an increasingly important commodity in our modern society. In the last thirty years, electricity usage has doubled to 26,615 terawatt hours (TWH).[2,3] The US alone makes up for 16.8% of this consumption. Recently, more resources have been invested into developing technologies to harvest renewable energy to handle this incoming crisis. Last year, the world produced 15.8% of its power through hydroelectric dams, 10.3% through nuclear power plants, and 9.1% through wind, solar, and other renewable sources.

But we still produce 64.2% of our power through the burning of fossil fuels. 17,086 TWH of electricity extracted from finite resources in our Earth: mainly coal, oil, and natural gas. This is completely unsustainable; we need new, outside of the box methods to produce electricity. Interestingly enough, we can find one from within… literally. The average human being consumes $8.36*10^6$ Joules every day. For the entire world that's $2.30*10^{19}$ joules per year, which is consequently released through physical movements and processes. And as it turns out, this energy equates to 6,383 TWH of electricity. To put that in perspective, that is the equivalent of burning **2.19 Trillion** pounds of coal. If we can harness a portion of this energy daily from a substantial number of people, we have the potential to produce significant amounts of clean power.

The optimal place to generate electricity from a lot of powerful people? The gym. While there has yet to be a fully commercialized system of generating energy from weightlifting machine, the physical concept has been used to generate power for years. Converting gravitational potential energy into rotational motion is the basis for hydroelectric power; even in ancient times, people used water wheels to grind wheat and other grains. The change in potential energy of bodies of water is still used today to create electrical power through dams, albeit much more efficiently. The energy creation process of these hydroelectric systems can be mimicked by adding an attachment on a weight machine's rotating pulley. This would be turned by the falling weights instead of falling water.

DESIGN APPROACH AND METHODS:

Holistic Design Goals

On the approach to this problem, the goal was to create reusable energy from exercise equipment. However, there were multiple subgoals and requirements that the design had to adhere to because of the human aspect.

The first was to not interfere with the user's workout; many gym-goers are very exact about their weights and reps. Any changes to the function of the machine would reduce the number of people who use it, and the energy production will decrease. To ensure usage, a clutch is required to control when the system engages and disengages. In this circumstance, the clutch must only be activated when the user is completely done with the set and the weight is in free-fall. To decide which type of clutch would be best in this situation, a design matrix is used. Velocity based activation is one of the key factors in this matrix, along with overall efficiency, efficiency at expected RPM, ability to manufacture, and simplicity. In this matrix, we compared a centrifugal clutch, ratcheting clutch, and velocity arm clutch, with the latter being a custom design proposed specifically for this system. The matrix showed the velocity arm clutch (to be referred to as VAC), as the most effective clutch for this design.

Clutch Design Matrix

	Overall Efficiency	Efficiency at Expected RPM	Simplicity	Ability to Manufacture	Velocity Based Activation	
Weight	90	60	40	50	60	**Totals**
Car Clutch	9 | 810	4 | 240	4 | 160	1 | 50	8 | 480	1740

Ratcheting Clutch	5 | 450	10 | 600	7 | 280	7 | 350	1 | 60	1740
Velocity Arm Clutch	8 | 720	9 | 540	5 | 200	6 | 300	9 | 540	**2300**

Another design goal was to slow down the falling weight stack in order to reduce noise and damage to the machine. The slamming of weights at the end of a set is a major issue for gym owners and users alike. Those who commit this act, colloquially known as 'Lunks', increase repair costs and intimidate other gym goers who are self-conscious about their own fitness. To solve this issue while optimizing electricity production some sort of flywheel system is needed. But this flywheel needs to be adaptable to many different weights, as a flywheel designed to slow down 100 lbs would not be effective at 20 lbs, and may even stall the machine. A design matrix shows that to maximize effectiveness and ease of manufacturing a variable inertia flywheel (VIF) should be used; a mechanism of custom design.

Flywheel Design Matrix

	Limits slamming of weights	Effective at all ranges of weights	Ease of Manufacture	Compact in size	
Weight	90	70	60	50	**Totals:**
No Flywheel	1 | 90	1 | 70	10 | 600	10 | 500	1260
Simple Flywheel	9 | 810	3 | 210	6 | 360	6 | 300	1680
Variable Inertia Flywheel	9 | 810	7 | 490	4 | 240	5 | 250	**1790**

The flywheel's creation is based on increasing the efficiency of the system by shifting the allocation of energy from the weight stack falling to the gearbox that drives the generator. A simple design that follows the same process is if there was only a mass attached to the pulley by a cord. The tangential velocities of both the mass and the pulley are the same if the cord connecting them is not slipping over the pulley. Therefore, if viewed from an energy perspective, increasing the moment of inertia of the pulley decreases the velocity of both systems, but increases the kinetic energy of the pulley rather than the kinetic energy of the mass. For an initial gravitational potential energy, the percentage of kinetic energy allocated to the pulley increases with a higher moment of inertia. This idea translates to the project very well. In order to increase the moment of inertia of the system, the most effective way was to use the idea of a flywheel.

A consideration that needs to be made is that for the same amount of torque, the acceleration is much higher for a system without a flywheel than a system with a flywheel. This can be determined using the Newtonian derivations rather than utilizing the work-kinetic energy theorem.

The original flywheel design also was based on allowing the gearbox to adapt to different weight stacks. For example, the gearbox's acceleration might be very small for a small weight stack if the flywheel has a moment of inertia based on a large weight stack. Thus, the idea of introducing a variable moment of inertia flywheel was created to combat this plan. The idea is that a continuous function based on the angular velocity could determine the position of masses inside of the flywheel. This would create an artificial transmission. The original gearbox that was designed around the motor's kv had a designated gear ratio for the position of the flywheel.

Through ideal calculations in MATLAB and experimental testing with the motor, it was deduced that a final gear ratio of 1:512 to the generator would be ideal, with the flywheel being geared up 1:32, thus limiting the final velocity of the weights to about 0.2 m/s. This final velocity was deemed to be satisfactory to prevent slamming of the weights. A copy of the MATLAB code can be seen in Appendix I.

Choosing the most efficient motor was a crucial aspect of the design process. We wanted to make sure that we would choose a motor that would be the most efficient above all else, so we made a design matrix based on that fact.

Generator Design Matrix

	Efficiency	Cost	Availability	
Weight	90	60	60	**Totals:**
Brushed DC motor	4 | 360	9 | 540	10 | 600	1500
Brushless DC motor	9 | 810	6 | 360	10 | 600	**1770**
Induction Motor	10 | 900	4 | 240	4 | 240	1380

Because of its efficiency and ease of access, we chose to use the Brushless DC motor (BLDC). When it comes to power generation by backdriving a DC motor, the most important thing to take into account is the motor's K_v, which is the angular velocity in revolutions per minute (rpm) the motor will rotate at when given 1 Volt. This constant is also reversible, in that if the motor was to spin at the motor's K_v, it would produce 1 Volt. For our purposes, we needed a motor with a small K_v so that it would require a low angular velocity to produce usable electricity.

One of the issues we faced when deciding on which BLDC to use was that there are a very small amount of commercially available BLDC's with low K_v. Because of this, we had to settle for a BLDC with a K_v of 800. This means that we would need a very high angular velocity to produce a usable voltage, hence our aggressive gear ratio. We encountered issues with this design later in our manufacturing process.

DESIGN DETAILS:

Since the project requires an attachable, adaptable system, the manufacturing method involved the design and creation of multiple components to be assembled in a modular fashion. As such, this section will detail their individual creation and then describe the actual assembly process.

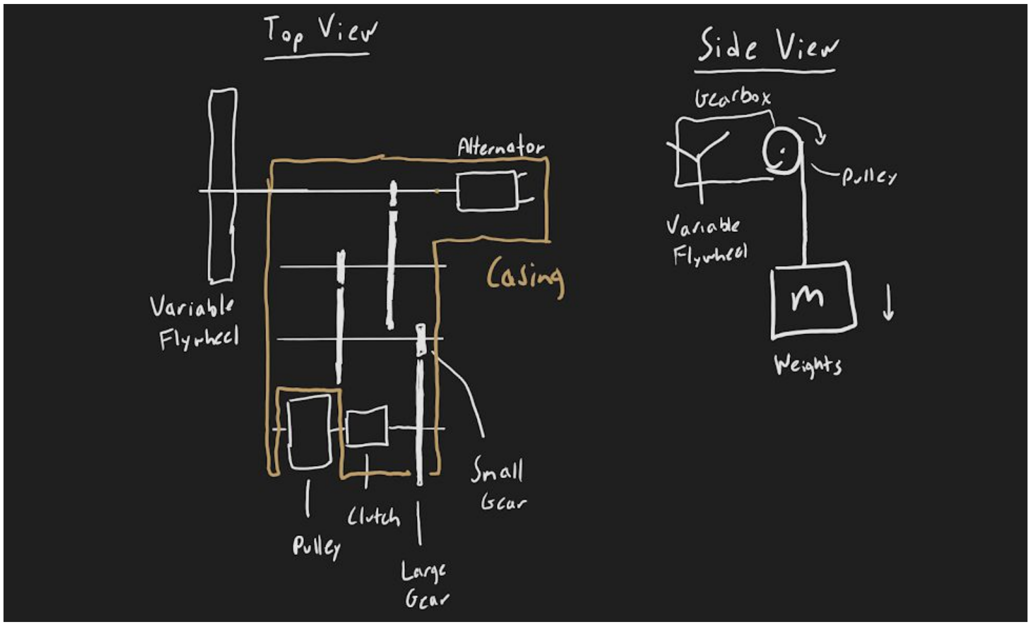


Figure 1: Rough sketch of plans for the full assembly

Velocity Activated Clutch (VAC):

The VAC activates in one direction at a defined, adjustable angular velocity. To do so, it relies on the principles of rotational physics and elastic extensions. In simplest terms, the VAC is a combination of a ratcheting clutch and a centrifugal clutch, substituting the frictional surface of a centrifugal clutch for a set of teeth. For the teeth to effectively connect with the outer ring of the clutch, the centrifugal clutch's bent arm is simulated with two arms that extend linearly. To control the rotational velocity at which these arms activate the outer ring, springs are attached (in this case, elastic bands are used as they follow similar laws but are easy to obtain). The relationship that allows us to utilize a VAC is shown below.

$$F_{spring} \propto F_{centripetal}$$

$$\Delta x_{spring} \propto \omega^2$$

The VAC was manufactured through the use of Solidworks and a 3D printer. It was created in three separate parts: the inner piece which rotates with the axle attached to the weightstack, 2 arms that extend from the middle to interact with the outside ring, and the outside ring which rotates the gearbox and motor. To control the Activation Velocity, a rubber band is looped through the arms with an elasticity tested in regards to the above proportionality statement..

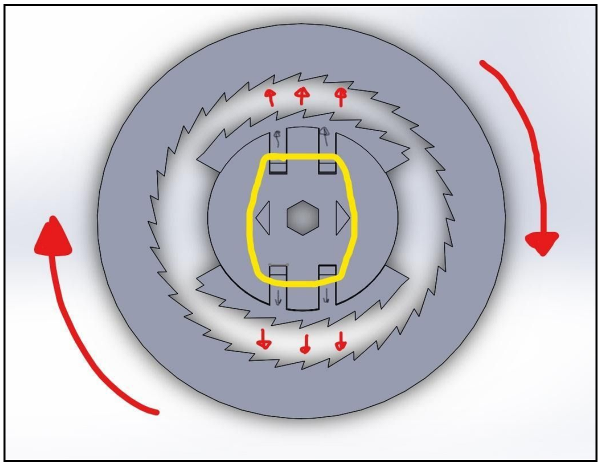


Figure 2a: CAD Velocity Activated Clutch

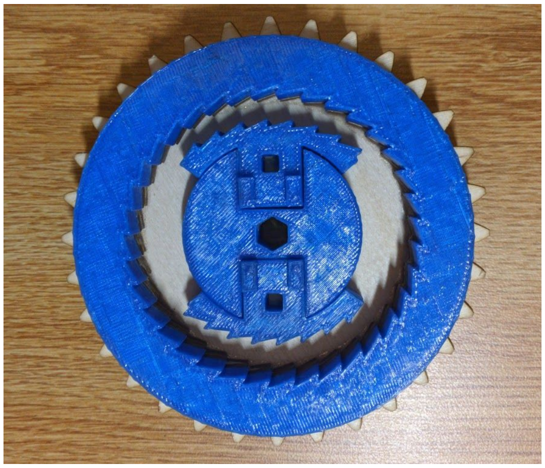


Figure 2b: Assembled VAC

Gearbox / Power Train:

The gearbox design planned has a 1:512 speeding up ratio from the machine's pulley to the generator shaft. As such, if the pulley were to be travelling at 1 rad/s, then the generator shaft would rotate at 512 rad/s in an ideal, frictionless environment. This gear ratio is attained through a combination of factors: pulley to roller, bevel gears, and laser-cut wooden gears. The pulley has a circumference four times larger than the roller it touches, so the pulley-roller system acts akin to a 1:4 gearing up. The bevel gears have a 1:2 ratio, speeding up the gears further. Finally, the entire laser-cut gear system acts as a 1:64 gear ratio, with three stages that are comprised of 1:4 gear ratios each. The annotated CAD can be seen in Figure 2:

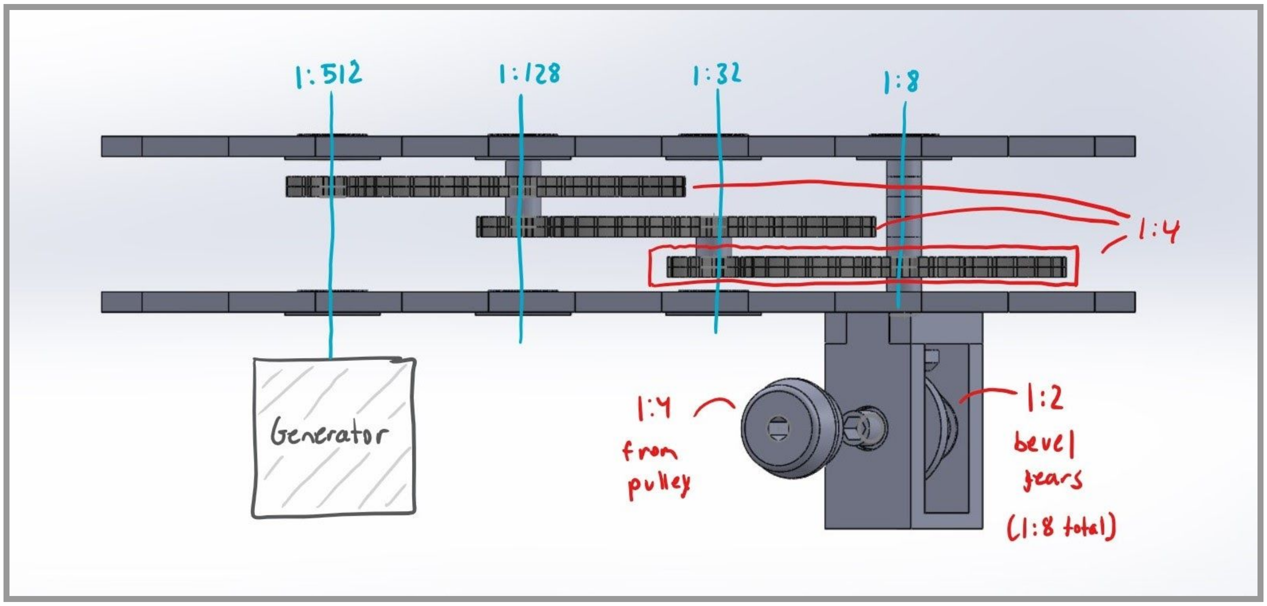


Figure 3: Annotated initial CAD of the gearbox system

All of the components of the gear train were either fabricated using rapid prototyping methods or bought off-the-shelf. We decided that to simplify the attachment of the gears to the shaft, we'd use 8mm hex shafts. Additionally, they fit into 8x22x7mm bearings easily (one of the most common types of bearing) and reduce, if not prevent, slippage of the gear from the axle through rotational shear stress. These axles were bought through goBILDA, an online vendor for aluminum and robotics parts. The bevel gearbox and its housing were 3D printed using PLA, as well as the roller that contacts the exercise equipment's pulley.

However, during testing, we began to encounter problems. The connectors in the bevel gear system snapped under the immense torque required to drive the gear ratio and we did not have time to print better replacements, so the bevel gear system was ditched. When testing the machine at Marino, we quickly realized that there was not enough friction between the roller and the equipment's pulley to drive the entire system. As such, we changed the system so that the cable rested on the roller, which can be seen in Figure 3. However, the roller was too weak to withstand the constant force of the cable and weights in the exercise machine, crumbling and snapping. In addition to unforeseen problems with the internal magnetic resistance of the back-driven motor (where it caused excessive static friction, preventing the gears from moving), we opted to decrease the gear ratio to 16:1 and use an arm-row approach to simulate the test.

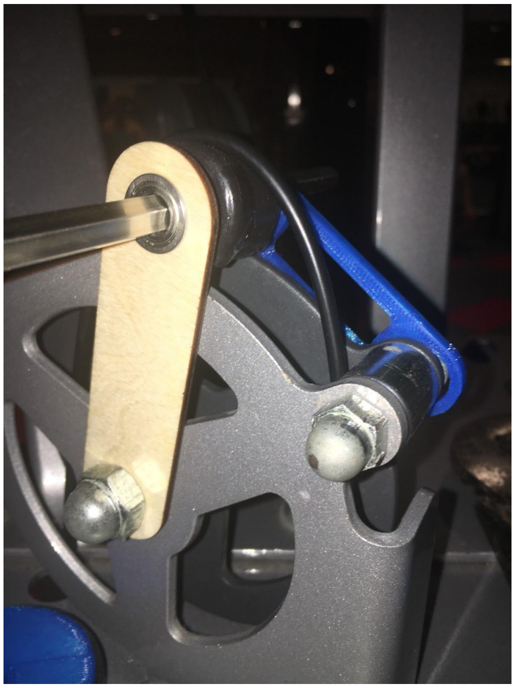


Figure 4: New system where the cable rested on the roller.

Variable Inertia Flywheel (VIF):

The VIF was first designed in SolidWorks and then manufactured using a combination of off-the-shelf parts and 3D printed parts. Initial designs of the flywheel were entirely 3D printed and consisted of two circular halves with slots for the masses. However, these were unable to be manufactured in the First Year Engineering Learning and Innovation Center (FYELIC) due to time, so we took an alternate route that required far less 3D printing. The final flywheel can be seen in Figure 4:

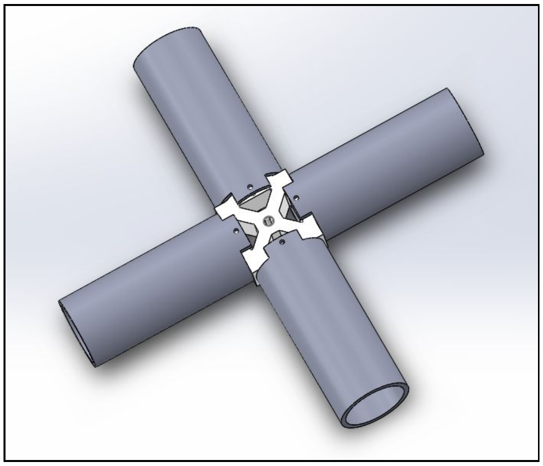


Figure 5a: CAD of the flywheel

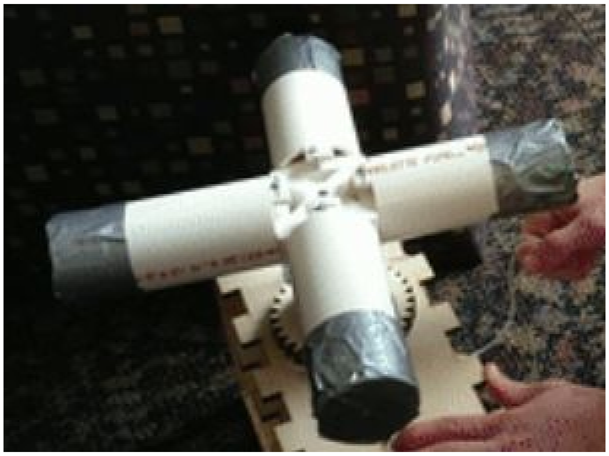


Figure 5b: Assembled flywheel

This final VIF was composed of four 2” diameter PVC pipes arranged in an “X” manner, rigidly attached to an ABS 3D printed part in the center with four 6-32 3” bolts. Inside each of the PVC pipes is a 500 gram mass that would be forced toward the center by a 10K N/m spring constant spring. However, due to a number of factors (including shipping times and practicality), we opted to forego implementation of the spring into the system. This resulted in a flywheel that had a constant moment of inertia. This does not degrade efficiency, but it makes certain loads on the gear box more efficient than others through the idea that more torque is produced with higher loads.

Generator and Other Electronics:

When backdriving a Brushless DC motor, they can be modeled by and are very similar in principle to 3-phase AC generators. As such, a 3-phase rectifier with smoothing capacitor circuit was required to convert the 3-phase AC signal to a constant DC signal. To do this, we created a circuit that converts the AC to DC, eliminates residual ripple voltage from signal, and reads the output voltage through the load (Figure 6).

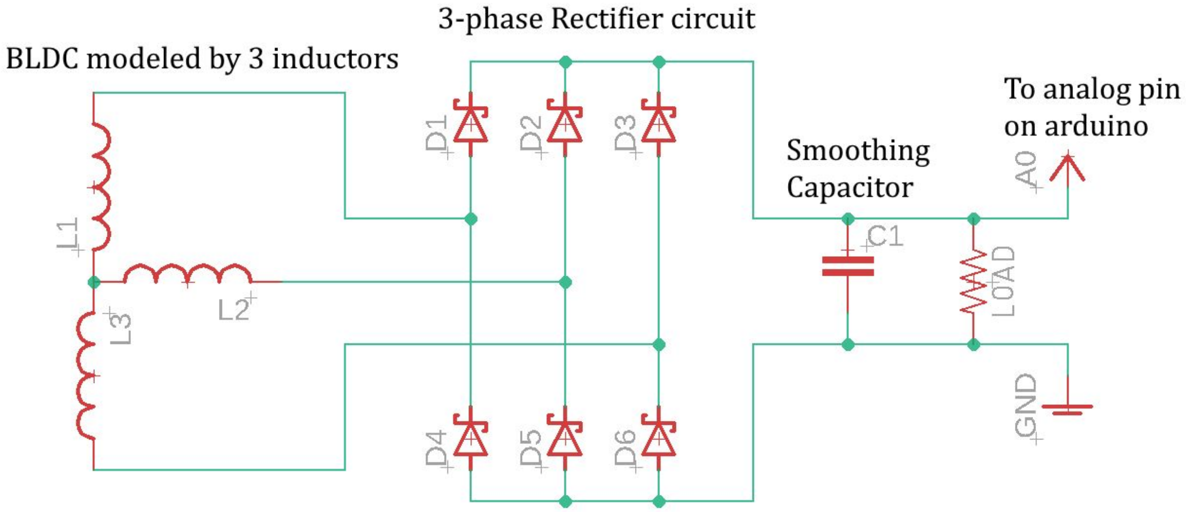


Figure 6: Annotated generation circuit design

To decide the resistance of the load, we tested resistors with values starting 1k ohm and worked our way down to two 1 ohm resistors in parallel. Such a low resistor value was utilized because we needed to maximize the power of the circuit. Power can be calculated knowing only voltage and resistance, so by keeping the resistance constant, power and voltage become directly related.

We found that by increasing the load, we drastically decreased the voltage drop across it. This did not prove to be an issue for us, but in the future it will be an important consideration.

A smooth DC signal would ensure accurate data collection with the arduino, so we utilized a smoothing capacitor. To calculate the value of this capacitor, we used the formula below:

$$V_{ripple} = \frac{I_{load}}{f \bullet C}$$

In order to minimize the ripple voltage, we used a 1000μF capacitor, which gave us a very smooth output signal.

Once the circuit was finalized, we created a custom PCB arduino shield that could be connected directly to the top of the RedBoard (Figure 7). When it arrived, surface mount diodes and LEDs were soldered to the board and data acquisition began.

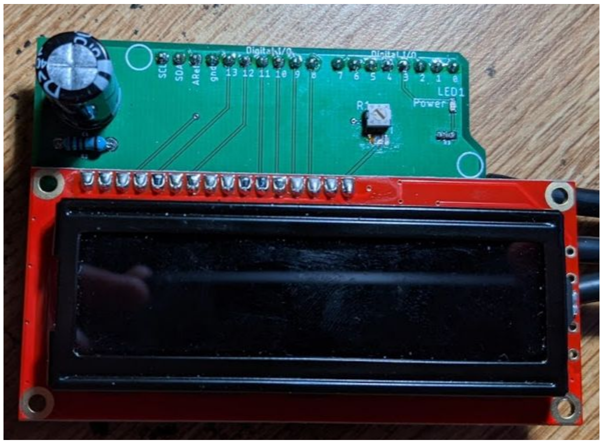


Figure 7a. front of custom PCB

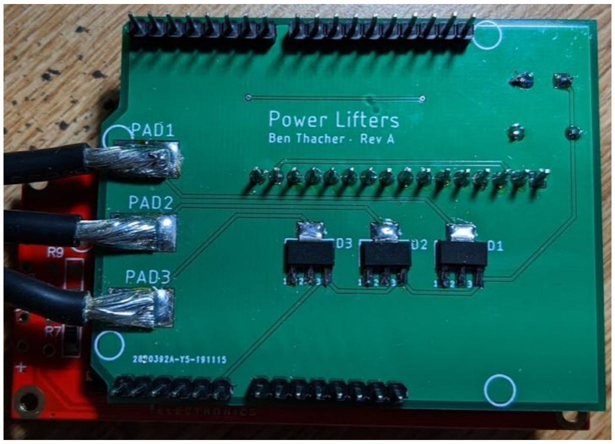


Figure 7b. back of custom PCB

Data Collection Process:

All of the hardware needed to calculate power output is found in our Arduino & generator system. While the Arduino can only measure voltage, that can be converted into current and power when that voltage is measured across a known resistance. The pertinent formulas are shown below.

$$I = \frac{V}{R}$$

$$P = \frac{V^2}{R}$$

The same program that displays maximum power, voltage, and current output on the lcd screen gathers information on the instantaneous power and accompanying timestamps. This data is sent to a Matlab Script where it can be further explored.

To collect applicable data, we simulated the use of a ‘row’ weightlifting machine by pulling a ripcord attached to the system. Because of the law of conservation of energy, the results from dropping a certain weight are exactly the same pulling a string with the same acceleration. To assure we measured the correct strength, we compared ripcord pulls with the lifting of actual free weights.

RESULTS:

To analyze performance at various weights, we tested three different force pulls, a low, medium, and high. We targeted these three strengths to mimic dropping 10 lbs, 20 lbs, and 45 lbs, respectively. Using MATLAB, we translated the Arduino data into instantaneous power and it’s integration, both in respect to time (See Appendix I for code & Figure 8 for graphs).

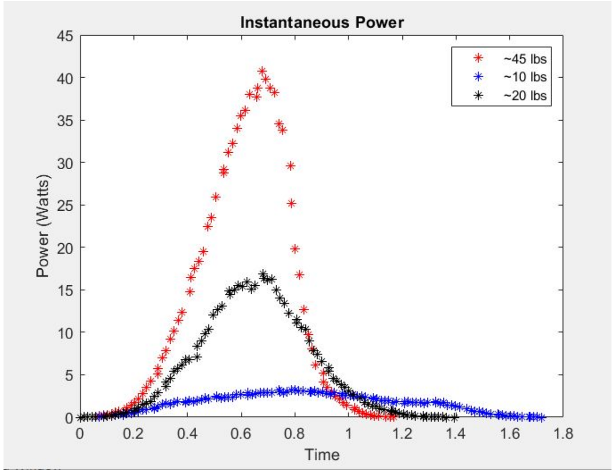


Figure 8a: Instantaneous Power Data

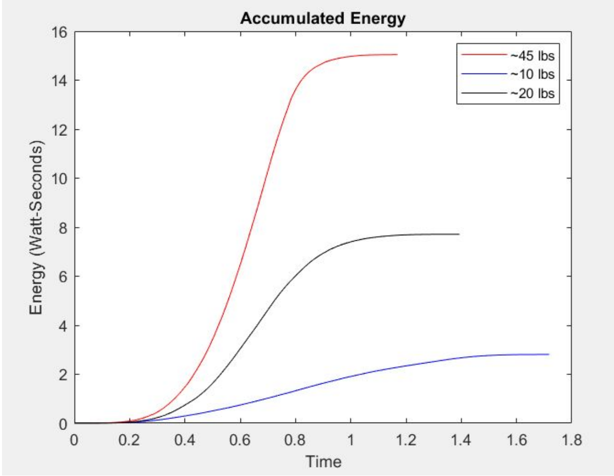
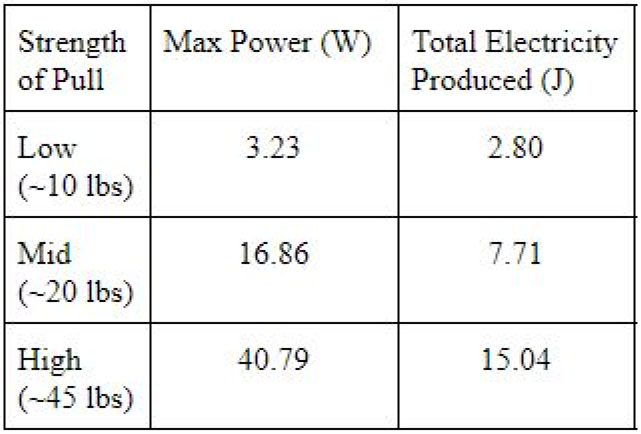


Figure 8b: Accumulated Energy Data

From our testing, we discovered that we could recover a notable amount of energy from each of our pulls. As seen in Table 1, the low pull had a maximum instantaneous power of 3.23 W and produced 2.80 J of electrical energy, the medium pull had a maximum instantaneous power of 16.86 W and produced 7.71 J of electrical energy, and the high pull had a maximum instantaneous power of 40.79 W and produced 15.04 J of electrical energy.



Strength of Pull	Max Power (W)	Total Electricity Produced (J)
Low (~10 lbs)	3.23	2.80
Mid (~20 lbs)	16.86	7.71
High (~45 lbs)	40.79	15.04

Table 1: Power and energy results

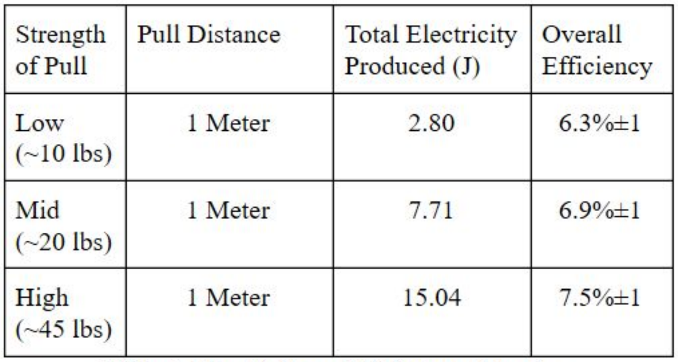
Keep in mind that the unit of accumulated energy (Watt-Seconds) is the electrical equivalent of Joules. Using this fact, we can calculate the efficiency of one pull.

$$Efficiency = \frac{\int P(t)dt}{KE}$$

To ensure an accurate simulation, the ripcord was pulled 1 meter for each load. To complete the efficiency calculation, we use the relation:

$$KE = PE = (mg)h$$

The estimated efficiency for a 45 pound drop is 7.5%. In this efficiency calculation, we added a tolerance of 1 percent as there may be a small variation in the strength of our pulls.



Strength of Pull	Pull Distance	Total Electricity Produced (J)	Overall Efficiency
Low (~10 lbs)	1 Meter	2.80	6.3%±1
Mid (~20 lbs)	1 Meter	7.71	6.9%±1
High (~45 lbs)	1 Meter	15.04	7.5%±1

Table 2: Electricity and Efficiency Results

DISCUSSION:

Tables 1&2 show that the system does manage to produce a significant amount of electricity in respect to the total potential energy of a weight machine. However, in its current state the project will not see much use from gym goers. Without being attached to a professionally manufactured machine, the ‘Power Lifter’ does not achieve its goal of producing power from an already existing machine. Instead, it exists only as a stand alone row machine. And while this can be operated as exercise equipment, very few people would choose to include it in their gym routine.

For the system to operate as planned, it requires a metal roller and gearbox to withstand the immense pressure; materials that are far beyond the scope of this project. Without proper manufacturing, its functionality dramatically decreases.

Despite its flaws, the ‘Power Lifter’ serves as a proof of concept of a more extensive idea: human-produced electricity. Hypothetically, a busy gym could possibly power itself through the sweat of its users. At Marino alone, 51 different machines could use our attachment if it were assembled professionally. And while it is difficult to predict the exact number of gym-goers per day, Marino’s near-constant stream of users would accumulate a great deal of electricity.

ETHICS:

The main consequence of the implementation of this device is enforcing the idea of being a “lunk” at the gym. For example, if a person becomes accustomed to dropping weights and goes to a different gym, the different gym might not have machines equipped with our regenerative device. They might continue dropping weights out of habit and be reprimanded by the gym staff or others using the gym for something that they did not intend to do.

However, this could also have an ethical benefit at gyms with our system. Organizations like Planet Fitness have built their businesses on the idea of inclusion and acceptance among gym-goers. For many people, the slamming of large weights from across the gym makes them feel inferior and self conscious, and often can prevent them from going to live a healthier lifestyle. Our system would help combat this, and make the gym a more wholesome environment for everyone involved.

CONCLUSION:

The production of energy from existing systems in weightlifting is not exactly the most prioritized issue in engineering. However, the creation of a system that serves that very purpose shows how engineering can allow humans to optimize every aspect of life to support each other. In this case, it was about creating power from an untapped source, like slamming weights at the gym. The results show that the 'Power Lifter' proof of concept can, with tweaks, be an effective commercial way of regenerating energy. Since the energy producing system was designed to be completely modular, with minimal adjustment, this concept could be used on any machine in the gym with a pulley attachment. For Marino Center at Northeastern University, there are approximately 51 machines that could utilize this system to take advantage of untapped energy. Not only this, but the system's energy production over time depends entirely on the traffic of the area, since it is based on human interaction. Since the gym is a high traffic area, this only boosts the ability of this concept to be enforced.

However, the progression of this energy concept to actual integration with everyday life would require a material revamp. This was not possible in the proof of concept due to various circumstances, such as money or manufacturing capability, but if implemented in a more fleshed out design, would allow the energy production to be even more efficient. Overall, the system succeeded in producing electrical energy from the wasted kinetic energy of weights slamming, and proved that this concept is not a fantasy. Optimization in the smallest ways matters in the largest of ways in the grand scheme of the planets energy consumption.

ACKNOWLEDGEMENTS:

We would like to thank Professor Bala Maheswaran for leading us through the engineering design process and helping us throughout our project, including providing us with more than enough FYELIC manufacturing vouchers. Also, we would like to thank the Northeastern Department of Physics for trusting our project and allowing us to borrow four expensive 500 gram masses. Lastly, we would like to thank Sarah Myles and the staff of Marino Recreation Center for allowing us to test on one of their machines.

REFERENCES:

1. "Alternators and Generators." Accessed December 5, 2019. https://www.allpar.com/eek/alternators.html.
2. BP. "BP Statistical Review of World Energy" 68 (2019). https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/pdfs/energy-economics/statistical-review/bp-stats-review-2019-full-report.pdf.
3. Enerdata. "Total Energy Consumption," 2018. https://yearbook.enerdata.net/total-energy/world-consumption-statistics.html
4. Hartman, Albert, and Wendy Lorimer. "Design Equations for BLDC Permanent Magnet Generators." Accessed December 5, 2019. http://performancemagnetics.com/images/BLDC_Generator_Design_Methodology_Performance_Magnetics_revI-2.pdf.
5. Learn Engineering. "How Does an Alternator Work ? - YouTube," 2014. https://www.youtube.com/watch?v=tiKH48EMgKE.
6. National Hydropower Association. "Hydropower Availability," n.d. https://www.hydro.org/waterpower/why-hydro/available/.
7. Thang, Nguyen Duc. "1700 Animated Mechanical Mechanisms," 2014. https://pwp.gatech.edu/mecapstone/wp-content/uploads/sites/334/2017/09/1700-Animated-Linkages.pdf.
8. Ven, James D. Van de. "Fluidic Variable Inertia Flywheel," 2015. http://me.umn.edu/~vandeven/ConfPaper-FluidicVIF_IECEC09.pdf.
9. Wikipedia. "Gear." Accessed December 5, 2019. https://en.wikipedia.org/wiki/Gear.
10. Wikipedia. "Transmission (Mechanics)." Accessed December 5, 2019. https://en.wikipedia.org/wiki/Transmission_(mechanics).
11. Wisconsin Valley Improvement Company. "How Hydropower Works," n.d. http://www.wvic.com/content/how_hydropower_works.cfm.

APPENDIX I: CODE

Arduino Code for Data Collection (not including miscellaneous LCD screen declarations):

```
void loop() {

  voltage = (analogRead(0) / 1023.0) * 5;

  if (voltage > highestVoltage)
          highestVoltage = voltage;
  if (millis() - start > 100) {
          lcd.clear();
          //print highest voltage
          lcd.setCursor(0, 0);
          lcd.print("V:");
          lcd.setCursor(3, 0);
          lcd.print(highestVoltage);

          //print highest power
          lcd.setCursor(0, 1);
          lcd.print("P:");
          lcd.setCursor(3, 1);
          lcd.print((highestVoltage * highestVoltage) / load);

          //print highest Current
          lcd.setCursor(10, 0);
          lcd.print("I:");
          lcd.setCursor(13, 0);
          lcd.print(highestVoltage / load);

          analogWrite(3, (int) (voltage * 255));
          start = millis();
  }

  energy=voltage*voltage/load ;
  Serial.print(micros());
  Serial.print(" ");
  Serial.println(energy);
}
```

MATLAB Code for Data Analysis:

```
data1=[First Data Set];
data1(:,1)=(data1(:,1)-data1(1,1))/1000000
times1=data1(:,1);
instantaneousPowers1=data1(:,2)
summ1=0;
accumulate1=[0];
for i=1:(length(data1)-1)
            summ1=summ1+(data1(i+1,1)-data1(i,1))*((data1(i+1,2)+data1(i,2))/2);
            accumulate1=[accumulate1,summ1];
end
figure(1)
plot(times1,instantaneousPowers1,'r*')
title("Instantaneous Power")
xlabel('Time')
ylabel('Power (Watts)')
hold on

figure(2)
plot(times1,accumulate1,'r-')
title("Accumulated Energy")
xlabel('Time')
ylabel('Energy (Watt-Seconds)')
hold on
data2=[Second Data Set];
data2(:,1)=(data2(:,1)-data2(1,1))/1000000
times2=data2(:,1);
instantaneousPowers2=data2(:,2)
summ2=0;
accumulate2=[0];
for i=1:(length(data2)-1)
            summ2=summ2+(data2(i+1,1)-data2(i,1))*((data2(i+1,2)+data2(i,2))/2);
            accumulate2=[accumulate2,summ2];
end
figure(1)
plot(times2,instantaneousPowers2,'b*')
title("Instantaneous Power")
xlabel('Time')
ylabel('Power (Watts)')

figure(2)
plot(times2,accumulate2,'b-')
title("Accumulated Energy")
xlabel('Time')
ylabel('Energy (Watt-Seconds)')

data3=[Third Data Set];
data3(:,1)=(data3(:,1)-data3(1,1))/1000000
times3=data3(:,1);
instantaneousPowers3=data3(:,2)
summ3=0;
accumulate3=[0];
for i=1:(length(data3)-1)
            summ3=summ3+(data3(i+1,1)-data3(i,1))*((data3(i+1,2)+data3(i,2))/2);
            accumulate3=[accumulate3,summ3];
end
figure(1)
plot(times3,instantaneousPowers3,'black*')
title("Instantaneous Power")
xlabel('Time')
ylabel('Power (Watts)')
legend('~45 lbs','~10 lbs','~20 lbs')

figure(2)
plot(times3,accumulate3,'black-')
title("Accumulated Energy")
xlabel('Time')
ylabel('Energy (Watt-Seconds)')
legend('~45 lbs','~10 lbs','~20 lbs')
```

MATLAB Code for Flywheel and Gearbox Optimization:

```
%% System calculations, assuming with gears and pulley and no friction

v_mass = input("Input velocity of the mass: ");

% General constants
g = 9.8;                                % in m/s^2
m = 18;                                 % in kg
h_max = 0.2;                            % in m

% Pulley constants
I_pulley = 0.000395555;                 % in kg*m^2
r_pulley = 0.1198;                      % in m
m_pulley = 0.21152;                     % in kg
w_p = v_mass / r_pulley;
rpm = w_p * 60/2/pi;

% Gear ratio (geared up) of the system (4 means 4:1), number of terms in
% matrix are the number of gears
gears = [4 4 4 4 4];
gb = zeros(1, size(gears, 2));          % matrix for gearbox, g(x) provides gear at specific stage
max_gear_ratio = 1;                     % has val of 1 temporarily until after for loop
for i = 1:size(gears, 2)
    max_gear_ratio = max_gear_ratio * gears(i);
    gb(i) = max_gear_ratio;
end
% MOI_gears = (10^-9) * [10000 10000];       % in g*mm^2
% MOI_transfer_gear = (10^-9) * 2500;        % in g*mm^2

% VMOI Variables
% Cylinder is the mass at the end of the spring
m_casing = 1;                           % in kg
m_cyl = .5;                             % in kg
m_all = m_casing + 4*m_cyl;             % in kg
r_fly = 0.15;                           % in m
k_spr = 15761;%LHL1250A                                 % in m
r_cyl = 0.0205;                         % in m
l_cyl = 0.0545;                         % in m
% Define energy terms
% Want everything in terms of w_p (to solve for w_p)
U_max_w = m * g * h_max;
U_h_w = 0;
K_trans_w = 0.5 * m * r_pulley^2 * w_p^2;
K_rot_pulley = 0.5 * I_pulley * w_p^2;
sumK_rot_gear = 1;
sumK_rot_tgear = 1;
K_rot_VMoI_fly = 0.5 * I_w_fly * (gb(2) * w_p)^2;


% Simpler equation, solves for w_p
% eqn_simple = U_max_w == U_h_w + K_trans_w + K_rot_pulley;
% S_simple = double(solve(eqn_simple));      % Solving for w_p
% w_p = S_simple(2);

% The gnarly equation
% eqn_complex = U_max_w == U_h_w  + K_trans_w + K_rot_pulley + sumK_rot_gear + sumK_rot_tgear + K_rot_VMoI_fly;
KGEARS = (.25*.029*.0016*(gb(1))^2)+(.25*.029*.0016*(gb(2))^2)+(.25*.029*.0016*(gb(3))^2)+(.25*.001*.0001*(gb(1))^2)+(.25*.001*.0001*(gb(2))^2)+(.25*.001*.0001*(gb(3))^2)
% VMOI Equation
eqn_vmoi = U_max_w == U_h_w + K_trans_w + K_rot_pulley + K_rot_VMoI_fly+KGEARS;
S_vmoi = double(solve(eqn_vmoi, I_w_fly));
fprintf('The required moment of inertia for the VMoI flywheel is %.4f \n', double(S_vmoi));
```